Appendix

Math Induction Proof

Suppose the cells in a grid contain symbols for n different jackpots ($n \geq 2$). Each cell contains exactly one symbol. The jackpots are called M_1, M_2, … , M_n. The number of cells that contain the symbol for M_i is a_i. The probability that a player will win the first jackpot is given by the following equation.

$P(M_1 < M_2 M_3 \dots M_n)$

$$= 1 - \left(\sum_{2 \le i \le n} \frac{a_1}{a_1 + a_i} - \sum_{2 \le i < j \le n} \frac{a_1}{a_1 + a_i + a_j} + \cdots + (-1)^n \frac{a_1}{a_1 + a_2 + \cdots + a_n} \right)$$

We will prove the preceding statement using the principle of math induction.

In this article, we have already verified the statement is true when $n = 2$ and $n = 3$. We will now show that if the preceding statement is true, then it is also true that

$P(M_1 < M_2 M_3 \dots M_n M_{n+1})$

$$= 1 - \left(\sum_{2 \le i \le n+1} \frac{a_1}{a_1 + a_i} - \sum_{2 \le i < j \le n+1} \frac{a_1}{a_1 + a_i + a_j} + \cdots + (-1)^{n+1} \frac{a_1}{a_1 + a_2 + \cdots + a_{n+1}} \right)$$

Assume that $P(M_1 < M_2 M_3 \dots M_n)$

$$= 1 - \left(\sum_{2 \le i \le n} \frac{a_1}{a_1 + a_i} - \sum_{2 \le i < j \le n} \frac{a_1}{a_1 + a_i + a_j} + \cdots + (-1)^n \frac{a_1}{a_1 + a_2 + \cdots + a_n} \right)$$

$P(M_1 < M_2 M_3 \dots M_{n+1})$

$$= 1 - P(M_2 < M_1 \text{ or } M_3 < M_1 \text{ or} \dots \text{or } M_{n+1} < M_1)$$

$$= 1 - P[(M_2 < M_1 \text{ or } M_3 < M_1 \text{ or} \dots \text{or } M_n < M_1) \text{ or } (M_{n+1} < M_1)]$$

$$= 1 - P(M_2 < M_1 \text{ or } M_3 < M_1 \text{ or} \dots \text{or } M_n < M_1) - P(M_{n+1} < M_1) + P[(M_2 < M_1 \text{ or } M_3 < M_1 \text{ or} \dots \text{or } M_n < M_1) \text{ and } (M_{n+1} < M_1)]$$

$$= 1 - P(M_2 < M_1 \text{ or } M_3 < M_1 \text{ or} \dots \text{or } M_n < M_1) - P(M_{n+1} < M_1) + P[(M_2 < M_1 \text{ and } M_{n+1} < M_1) \text{ or } (M_3 < M_1 \text{ and } M_{n+1} < M_1) \text{ or} \dots \text{or } (M_n < M_1 \text{ and } M_{n+1} < M_1)]$$

$$= 1 - [1 - P(M_1 < M_2 M_3 \dots M_n)] - P(M_{n+1} < M_1) + P[(M_2 < M_1 \text{ and } M_{n+1} < M_1) \text{ or } (M_3 < M_1 \text{ and } M_{n+1} < M_1) \text{ or} \dots \text{or } (M_n < M_1 \text{ and } M_{n+1} < M_1)]$$

$$= P(M_1 < M_2 M_3 \dots M_n) - P(M_{n+1} < M_1) + P[(M_2 < M_1 \text{ and } M_{n+1} < M_1) \text{ or } (M_3 < M_1 \text{ and } M_{n+1} < M_1) \text{ or} \dots \text{or } (M_n < M_1 \text{ and } M_{n+1} < M_1)]$$

$$= 1 - \left(\sum_{2 \le i \le n} \frac{a_1}{a_1 + a_i} - \sum_{2 \le i < j \le n} \frac{a_1}{a_1 + a_i + a_j} + \cdots + (-1)^n \frac{a_1}{a_1 + a_2 + \cdots + a_n} \right) - \frac{a_1}{a_1 + a_{n+1}}$$

$$+\sum_{2\le i\le n}\frac{a_1}{a_1+a_i+a_{n+1}}-\sum_{2\le i\le j\le n}\frac{a_1}{a_1+a_i+a_j+a_{n+1}}+\cdots+(-1)^n\frac{a_1}{a_1+a_2+\cdots+a_{n+1}}$$

$$=1-\left(\left(\sum_{2\le i\le n}\frac{a_1}{a_1+a_i}\right)+\frac{a_1}{a_1+a_{n+1}}-\sum_{2\le i<j\le n}\frac{a_1}{a_1+a_i+a_j}-\sum_{2\le i\le n}\frac{a_1}{a_1+a_i+a_{n+1}}+\cdots\right.$$
$$\left.+(-1)^{n+1}\frac{a_1}{a_1+a_2+\cdots+a_{n+1}}\right)$$

$$=1-\left(\sum_{2\le i\le n+1}\frac{a_1}{a_1+a_i}-\sum_{2\le i<j\le n+1}\frac{a_1}{a_1+a_i+a_j}+\cdots+(-1)^{n+1}\frac{a_1}{a_1+a_2+\cdots+a_{n+1}}\right)$$