# Some comments on 4×4 philatelic Latin squares

Peter D. Loly and George P. H. Styan

compiled: November 24, 2009

Postage stamps are occasionally issued in sheetlets of $n$ different stamps printed in an $n \times n$ array containing $n$ of each of the $n$ stamps. Sometimes the $n \times n$ array forms what we call a *philatelic Latin square* (PLS): each of the $n$ stamps appears exactly once in each row and exactly once in each column. In Figure 1 we display such a sheetlet (in full with selvage) with $n = 4$.


Figure 1. Four international scientific congresses: Canada 1972, PLS type $a324$.

In July–August 1972, Canada hosted four international congresses concerned with the exploration and development of the earth and man's activities on the planet. Featured on the stamps in Figure 1 (top row, left to right) are: (1) aerial map photography, for the 12th Congress of the International Society of Photogrammetry, (2) contour lines, the 6th International Conference of the International Cartographic Association, (3) a geological fault (cross-section of the crust of the earth, showing different layers of material), for the 24th International Geological Congress, (4) aerial view, for the 22nd International Geographical Congress.

## The 4×4 Latin Squares

There are just four $4 \times 4$ Latin square matrices in *reduced-form,* i.e., 1234 in the top row and first column:

$$\mathbf{L}_a = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 1 \\ 3 & 4 & 1 & 2 \\ 4 & 1 & 2 & 3 \end{pmatrix}, \quad \mathbf{L}_b = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 4 & 1 & 2 \\ 4 & 3 & 2 & 1 \end{pmatrix}, \tag{1}$$

$$\mathbf{L}_c = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 1 & 3 \\ 3 & 1 & 4 & 2 \\ 4 & 3 & 2 & 1 \end{pmatrix}, \quad \mathbf{L}_d = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 2 & 4 & 1 & 3 \\ 3 & 1 & 4 & 2 \\ 4 & 3 & 2 & 1 \end{pmatrix}. \tag{2}$$

Permuting the last three rows of each of the four Latin square matrices $\mathbf{L}_a, \mathbf{L}_b, \mathbf{L}_c, \mathbf{L}_d$ generates 6 Latin squares and so there are 24 *standard-form* $4 \times 4$ Latin squares in all. By standard-form we mean that 1234 is in the top row but not necessarily in the first column. We will refer to these 24 standard-form Latin squares with a letter $a, b, c, d$ and three numbers as they appear in column 1, rows 2,3,4. Thus the PLS in Figure 1 is of type $a324$ and may be represented by

$$\mathbf{L}_a^{(324)} = \begin{pmatrix} 1 & 2 & 3 & 4 \\ 3 & 4 & 1 & 2 \\ 2 & 3 & 4 & 1 \\ 4 & 1 & 2 & 3 \end{pmatrix}. \tag{3}$$

We note that $\mathbf{L}_a^{(324)}$ and the array of 16 stamps in Figure 1 are in *Sudoku form* in that the four stamps each appear once in the top left, top right, bottom left, and bottom right $2 \times 2$ corners, and so the full Latin square is the solution to a *mini-Sudoku* puzzle; Yates [37, p. 455] says that such Latin squares have "balanced corners".

## Latin Squares and Experimental Design

We were surprised to find that $4 \times 4$ Latin square designs have been used in printing postage stamps, with the first such use apparently being in 1972 (by Canada, Figure 1). Latin square designs of size $4 \times 4$ have been used in Europe at least since the 13th century, when the Majorcan writer and philosopher Ramon Llull (1232–1316) published his "First elemental figure" in his *Ars Demonstrativa* in 1283.

There is a huge literature on Latin squares, see, e.g., [1, 5, 9, 23, 29, 32]. It was observed by Ritter [23] that:

> It seems that Latin squares were originally mathematical curiosities, but serious statistical applications were found early in the 20th century, as experimental designs. The classic example is the use of a Latin square configuration to place 3 or 4 different grain varieties in test patches. Having multiple patches for each variety helps to minimize localized soil effects. Similar statements can be made about medical treatments.

Laywine & Mullen [15, ch. 12, p. 188] noted

> While the study of Latin squares is usually traced to a famous problem involving 36 officers considered in 1779 [12] by Leonhard Euler (1707–1783), statistics provided a major motivation for important work in the early and middle decades of the 20th century. Indeed many of the most important combinatorial results pertaining to Latin squares were obtained by researchers whose primary self-description would be that of a statistician.

Indeed it seems that the name Latin squares is due to Euler, who used Latin characters as symbols [36]; we have found six stamps [29] issued in his honor.

In the Second Edition of their *Handbook of Combinatorial Designs*, Colbourn & Dinitz [5, p. 21] identify, in their timeline of the main contributors to design theory from antiquity to 1950 (consecutively, born between 1890 and 1902):

Sir Ronald Aylmer Fisher (1890–1962),
William John Youden (1900–1971),
Raj Chandra Bose (1901–1987),
Frank Yates (1902–1994).

Latin squares have, however, been used in the statistical design of experiments for over 200 years. In randomized controlled trials, often there are too many treatments to cross to have replicates in each factorial treatment combination. The idea of Latin square designs allows main effects to be estimated and tested for significance. The general idea of factorial design with replication, though, has also been important in scientific research and in the history of statistics.

As described by Preece in his excellent survey on "R. A. Fisher and experimental design: a review" [19], probably the earliest use of a Latin square in an experimental design was by the agronomist François Cretté de Palluel (1741–1798), who in 1788 published a study [6, 7, 28] of an experiment involving the fattening of 16 sheep in France, four of each of four different breeds. The advantage of using a Latin-square design in this experiment was that only 16 sheep were needed rather than 64 for a completely cross-classified design. The purpose here was to show that one might just as well feed sheep on root vegetables during the winter; this was much cheaper, and easier, than the normal diet of corn and hay. For further details see the 1999 article in *Chance* by Ullrich [33], and the more recent publications [1, 4, 30, 32, 34]; see also [20, 21, 22].

## Latin Squares and Postage Stamps

In this article we present examples (Table 1) of 10 different types of $4 \times 4$ philatelic Latin squares from 10 different countries: Albania, Canada, Gambia, Guinea, Guinea-Bissau, Hong Kong (China), Malawi, Pitcairn Islands, Portugal, Tristan da Cunha from 1972–2009. The stamps feature several different topics: animals, artworks, birds, Mickey Mouse, and scientific congresses. As the mathematics educator William Leonard Schaaf (1898–1992) wrote in his 1978 book [25, p. xiii] "The postage stamps of the world are, in effect, a mirror of civilization."

Table 1. Standard-form $4 \times 4$ Latin squares and PLS counts and examples.

| PLS type | Sudoku form | special | special | # PLS identified | WWF | Macau | Figure | topic | country | year | WWF |
|---|---|---|---|---|---|---|---|---|---|---|---|
| *a*234 | | | | 54 | 43 | 2 | 2 | Mickey Mouse | Albania | 1999 | 0 |
| *a*324 | 1 | block-Latin | backwards circulant | 3 | 2 | 0 | 1 | scientific congresses | Canada | 1972 | 0 |
| *a*342 | 1 | | | 1 | 1 | 0 | 3 | subantarctic fur seals | Tristan da Cunha | 2004 | 1 |
| *a*432 | | block-Latin | forwards circulant | 36 | 26 | 0 | 4 | Lilian's lovebirds | Malawi | 2009 | 1 |
| *b*324 | 1 | centro-symmetric | | 51 | 13 | 29 | 5 | art works | Hong Kong | 2002 | 0 |
| *b*342 | 1 | | | 1 | 1 | 0 | 6 | African buffalo | Guinea-Bissau | 2002 | 1 |
| *b*423 | 1 | | Hudson systematic | 8 | 2 | 0 | 7 | Pyrenean desman | Portugal | 1997 | 1 |
| *b*432 | 1 | block-Latin | | 3 | 1 | 0 | 8 | black-crowned crane | Gambia | 2006 | 1 |
| *c*342 | | | | 1 | 1 | 0 | 9 | baboons & mangabeys | Guinea | 2000 | 1 |
| *d*324 | | centro-symmetric | | 1 | 1 | 0 | 10 | terns & noddies | Pitcairn Islands | 2007 | 1 |
| total | 6 | | | 159 | 91 | 31 | | | | | 7 |

As of November 24, 2009, we have identified 159 examples of PLS from 75 countries (stamp-issuing authorities) but only PLS of these 10 types of $4 \times 4$ standard-form Latin squares:

$$a234, a324, a342, a432, b324, b342, b423, b432, c423, d324;$$

see Table 1. We have, however, found $4 \times 4$ PLS of these 5 types $a243, a423, c324, d243, d342$ embedded in sheets of more than 16 stamps and we will discuss these in a further article, in which we will also consider PLS of sizes $2 \times 2$, $3 \times 3$ and $5 \times 5$. We would be pleased to discover $4 \times 4$ PLS of types

$$b234, b243, c234, c243, c342, c432, d234, d423, d432.$$

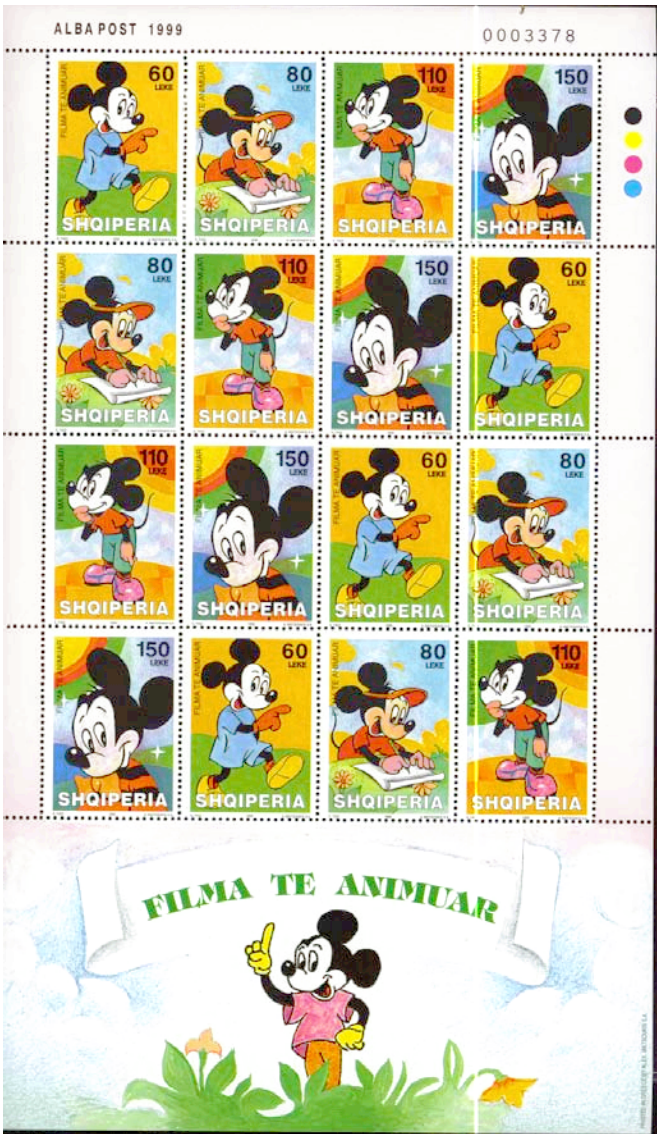
Of these 159 PLS examples, 91 were issued for the World Wide Fund for Nature (WWF) from 1988–2009 [14] and 31 from Macau. The WWF is an international non-governmental organization working on issues regarding the conservation, research and restoration of the environment, formerly named the World Wildlife Fund, which remains its official name in the United States and Canada.

## Philatelic Latin Squares of type *a*

The stamps shown in Figure 2 form a PLS of type $a234$, which is a one-step backwards circulant, and is $2 \times 2$ block-Latin in that the top left $2 \times 2$ corner of four stamps is the same as that in the bottom right corner, and the top right corner of four stamps is the same as that in the bottom left corner. This seems to be the most popular type of PLS—we have found 52 examples (Table 1). The stamps were issued by Albania in 1999 and feature Mickey Mouse, the celebrated animated character that became an icon for The Walt Disney Company. Mickey Mouse was created in 1928 by the well-known American film producer Walter Elias "Walt" Disney (1901–1966) and the American cartoonist Ub Iwerks (1901–1971).

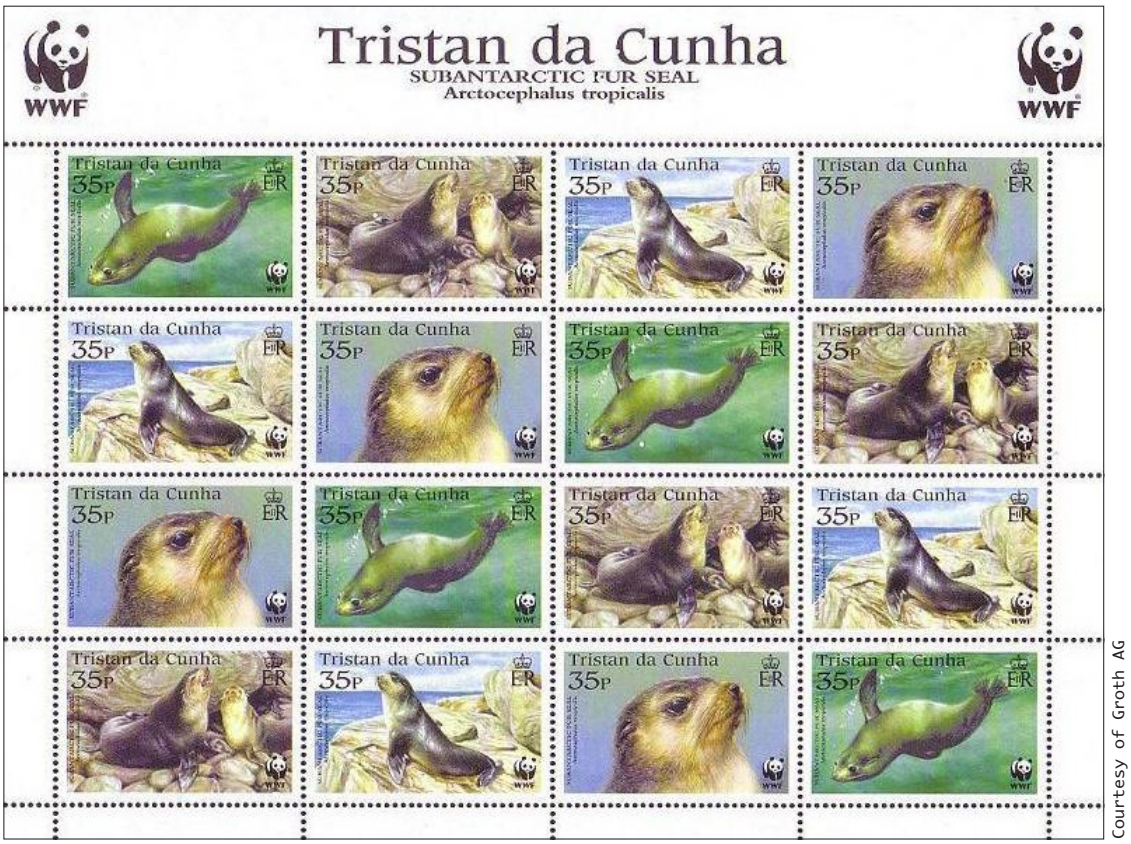
The only PLS we have found of type $a342$ is shown in Figure 3—it is also in Sudoku form like the PLS of type $a324$ shown in Figure 1. The stamps were issued in 2004 by Tristan da Cunha, a group of remote volcanic islands in the south Atlantic Ocean, 1,750 miles from South Africa and 2,088 miles from South America and the most remote inhabited archipelago in the world. Tristan da Cunha is a dependency of the British overseas territory of Saint Helena, 1,510 miles to the north. Depicted in Figure 3 is the sub-antarctic fur seal (*Arctocephalus tropicalis*), which is found in the southern parts of the Indian and Atlantic Oceans.

The stamps shown in Figure 4 form a PLS of type $a432$, which is a one-step forwards circulant and is $2 \times 2$ block-Latin. This is a popular type of PLS—we have found 36 examples (Table 1). The stamps shown in Figure 4 were issued in 2009 by Malawi and feature Lilian's lovebird (*Agapornis lilianae*), a small African parrot species. The Republic of Malawi is a landlocked country in southeast Africa formerly known as Nyasaland. The Scottish explorer David Livingstone (1813–1873) reached Lake Malawi (then Lake Nyasa) in 1859, and the British colonial government of Nyasaland was formed in 1891.



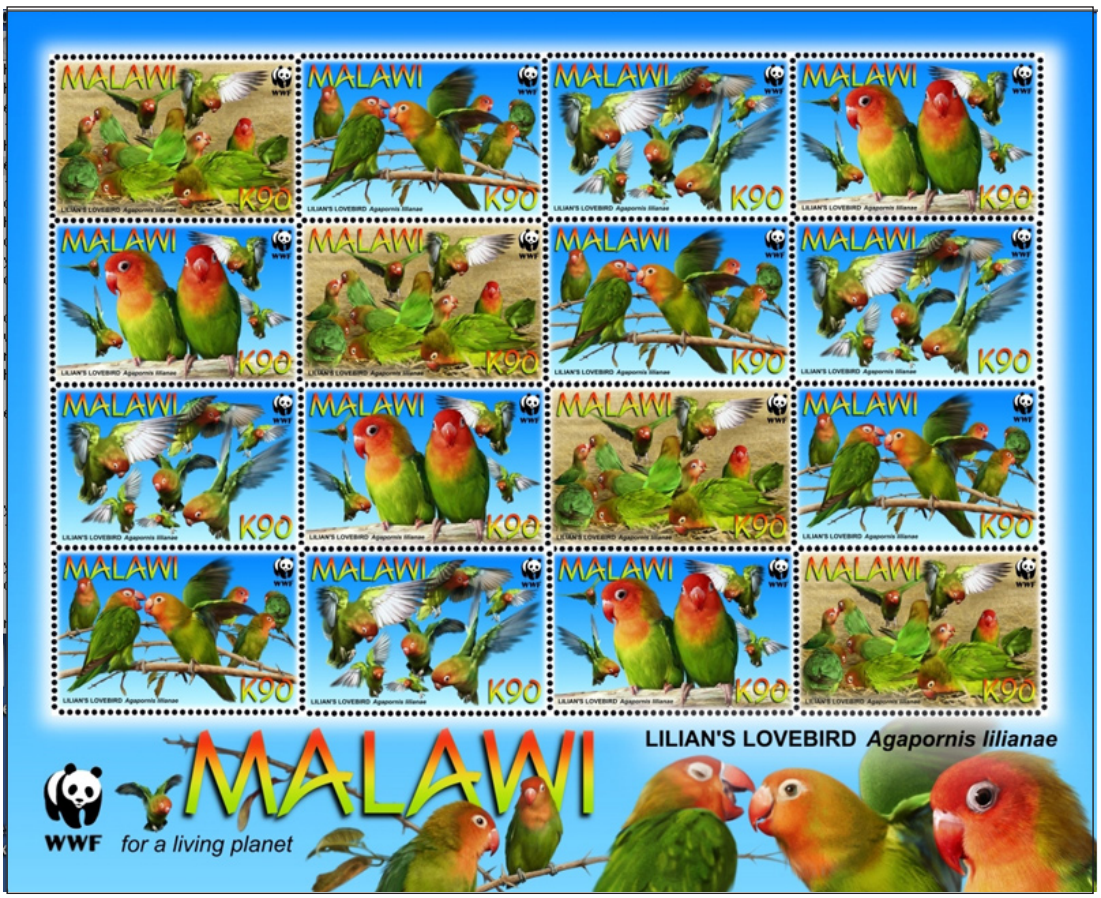

Courtesy of George P. H. Styan

Figure 2. Mickey Mouse: Albania 1999, PLS type $a234$.




Courtesy of Groth AG

Figure 3. Subantarctic fur seal: Tristan da Cunha 2004, PLS type $a342$.




Courtesy of Groth AG

Figure 4. Lilian's lovebird: Malawi 2009, PLS type $a432$.

## Philatelic Latin Squares of type *b*

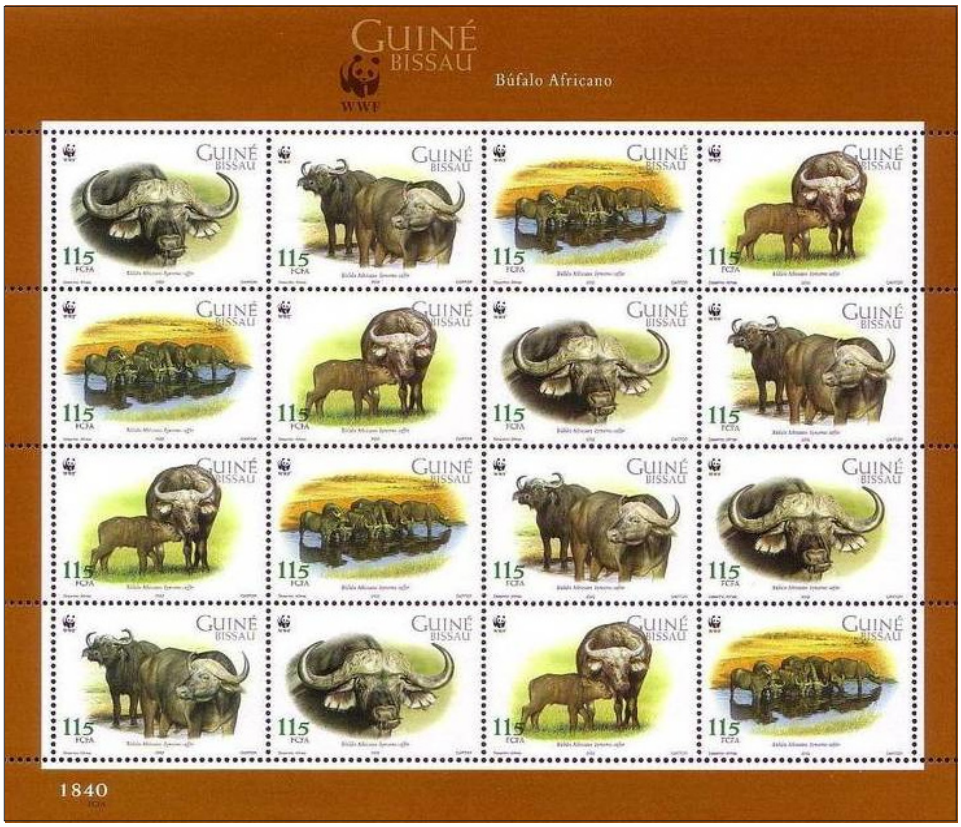

Courtesy of George P. H. Styan

Figure 5. Hong Kong Art Collections: Hong Kong, China 2002, PLS type $b324$.

The stamps shown in Figure 5 form a PLS of type $b324$, which is in Sudoku form and another popular type of PLS—we have found 45 examples (Table 1), many from Macau and from Hong Kong. The stamps shown in Figure 5 were issued by Hong Kong, China, in 2002 and feature four works of art: (top row, left to right) "Lines in Motion" by Chui Tze-hung, "Volume and Time" by Hon Chi-fun, "Bright Sun" by Aries Lee, and "Midsummer," by Irene Chou. Hong Kong, which was a British Crown colony until 1997, is now a Special Administrative Region of the People's Republic of China but continues to issue its own postage stamps.

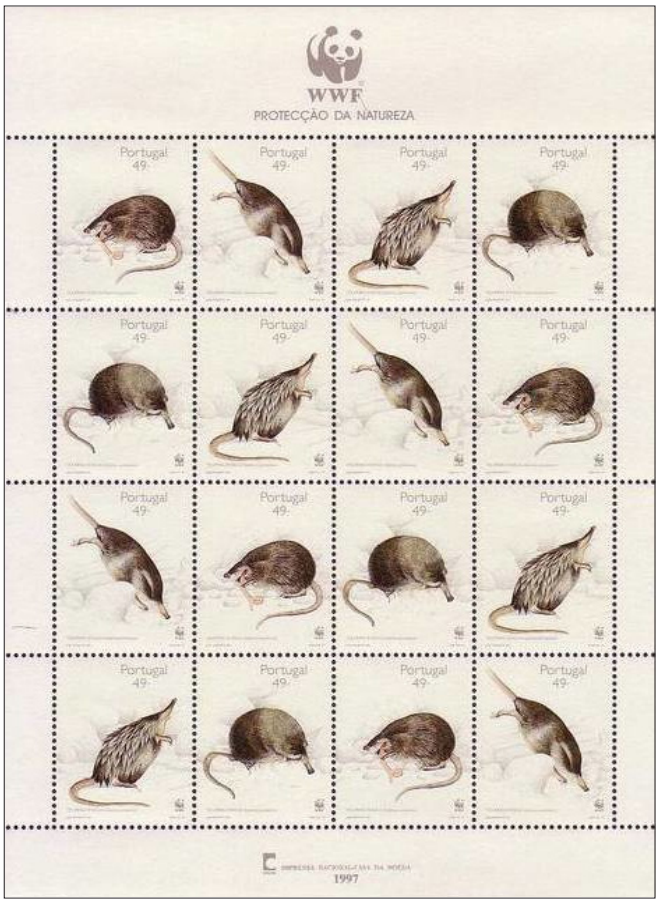
The sheetlet shown in Figure 6 comes from Guinea-Bissau (*Guiné-Bissau*), in western Africa; formerly Portuguese Guinea, upon independence, it added the name of its capital, Bissau, to the country's name in order to prevent confusion between itself and the nearby Republic of Guinea (*République de Guinée*), which was formerly French Guinea (and which issued the stamps shown in Figure 9). Depicted is the African buffalo (*Syncerus caffer*), a large African cloven-hoofed mammal. This sheetlet is the only PLS of type $b342$ that we have identified.

The stamps shown in Figure 7 were issued by Portugal in 1997 and depicts the Pyrenean desman (*Galemys pyrenaicus*), a small semi-aquatic mammal that lives in the Iberian peninsula; its body is like that of a muskrat, its nose like that of a hedgehog and its feet like that of a duck-billed platypus. The desman uses its snout as a periscope to breathe above the water's surface. The stamps form a PLS of type $b423$, which is in Sudoku form.




Courtesy of Groth AG

Figure 6. African buffalo: Guinea-Bissau 2002, PLS type $b342$.




Courtesy of Groth AG

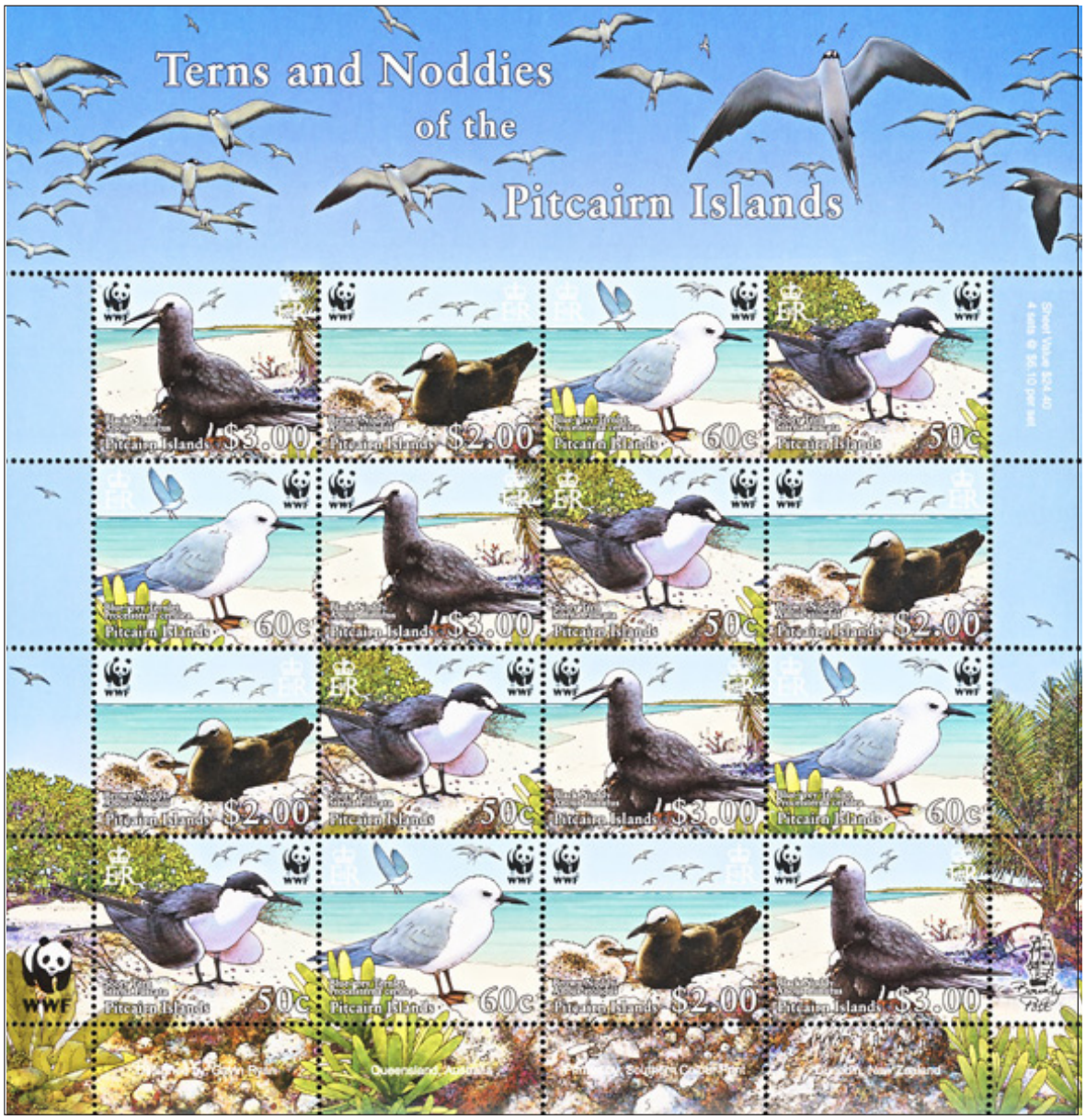
Figure 7. Pyrenean desman: Portugal 1997, PLS type $b423$.


Courtesy of George P. H. Styan

Figure 8. Black crowned crane: Gambia 2006, PLS type $b432$.


Courtesy of Groth AG

Figure 9. Baboons & mangabeys: Guinea 2000, PLS type $c423$.




Courtesy of George P. H. Styan

Figure 10. Terns & noddies: Pitcairn Islands 2007, PLS type $d324$.

The sheetlet shown in Figure 8 forms a PLS of type $b432$, which is in Sudoku form and is block-Latin. The sheetlet comes from the Republic of The Gambia, commonly known as Gambia, the smallest country on the African continental mainland. It is bordered to the north, east, and south by Senegal, and has a small coast on the Atlantic Ocean in the west. Depicted is the Black Crowned Crane (*Balearica pavonina*), a bird in the family *Gruidae.*

## Philatelic Latin Squares of types *c* and *d*

The PLS of type $c423$ (Figure 9) comes from the Republic of Guinea (*République de Guinée*) and features the Guinea Baboon (*Papio papio*) in row 1, columns 1 and 2, and the Collared Mangabey (*Cercocebus torquatus*) in row 1, columns 3 and 4. The Guinea Baboon is a baboon from the Old World monkey family which inhabits a small area in western Africa. It has reddish brown hair and a hairless, dark-violet or black face, which is surrounded by a small mane.

The Collared Mangabey, also known as the Red-capped Mangabey, White-collared Mangabey (leading to easy confusion with *Cercocebus atys lunulatus*), or Cherry-crowned Mangabey, is a species of primate in the *Cercopithecidae* family; it is a long-tailed and long-legged, relatively large monkey of gracile form with conspicuous white eyelids and a mat of short, backwardly directed hairs on the crown.

Our last example of a PLS is of type $d324$ (Figure 10) and the only PLS of this type that we have found; we call this type *criss-cross* since the same stamp appears in each cell in the main forwards diagonal, and the same stamp appears in each cell in the main backwards diagonal. The stamps were issued by the Pitcairn Islands, a British overseas territory (formerly a British colony), the last remaining in the Pacific. A group of four volcanic islands in the southern Pacific Ocean, the Pitcairn Islands are best known as the home of the descendants of the Bounty mutineers and the Tahitians who accompanied them, an event retold in numerous books

and films.

Shown in row 1, column 1, is the Black Noddy (*Anous minutus*), a seabird of the tern family. It resembles the closely related Brown Noddy (*Anous stolidus*) in row 1, column 2, but is smaller with darker plumage, a whiter cap, a longer, straighter beak and shorter tail. The Blue-grey Ternlet (*Procelsterna cerulea*) in row 1, column 3, and the Sooty Tern (*Sterna fuscata* in row 1, column 4, are seabirds of the tern family (*Sternidae*).

## Some Concluding Remarks

We do not know if the ten Latin square designs that we have found used in printing postage stamps are also the ten most popular for research design purposes, or if they are particularly advantageous relative to other possibilities. We note, however that Latin squares of types $b324, b342, b423, b432$ all have "balanced corners" (which we call Sudoku form) and are such that the treatment degrees of freedom can be partitioned into two from the first group of contrasts and one from the second [37, p. 455]; moreover $b423$ is called "Hudson's systematic square" [37, Fig. 4, p. 453]: Abram Wilfrid Hudson (1896–1982) was a crop experimentalist with the New Zealand Department of Agriculture who conducted an extensive correspondence [27] with "Student" (William Sealy Gosset, 1876–1937); this correspondence mainly concerned randomization, a practice which Hudson unwaveringly opposed [24, p. 154].

Hudson's "balanced Latin squares" $b324, b342, b423$ define magic squares in that in addition to the numbers in the rows and columns all adding to 10 so do the numbers in the main forwards and backwards diagonals; the only other $4 \times 4$ standard-form Latin square that has this property is type $d234$, given by the matrix $\mathbf{L}_d$ in (2) at the beginning of this article; we have not found a philatelic Latin square for type $d234$.

The type $b324$ (Figure 5) is the second most popular PLS and in addition to being in Sudoku form is also centrosymmetric in that flipping the rows and then flipping the columns yields the original design. In fact $b324$ is the only type of $4 \times 4$ Latin square in standard form, that is both centrosymmetric and is in Sudoku form; the type $d234$ is also centrosymmetric but is not in Sudoku form. In a follow-up article, planned for the next issue of *Chance,* we will look at $2 \times 2$, $3 \times 3$, and $5 \times 5$ philatelic Latin squares.

## Acknowledgements

We are most grateful to Michael D. Larsen, Executive Editor of *Chance*, for inviting us to prepare this article and for his most helpful comments on the first draft of this article, which is based, in part, on the research report by Loly & Styan [17]. Many thanks to Nicolas Ammerlaan, Oskar Maria Baksalary, Christian Boyer, Catherine Cameron, Ka Lok Chu, Harold V. Henderson, Jeffrey J. Hunter, Ian D. Kimmerly, Alexander Kovačec, Sid Kroker, Simo Puntanen, Daniel J. H. Rosenthal, Evelyn Matheson Styan, and Götz Trenkler for their help.

The discovery of our first philatelic Latin square (Figure 1) was at an auction at the Winnipeg Philatelic Society. Information about philatelic Latin squares may be found in the excellent open-access websites of Baker [2], BiStamp.com [3], delcampe.com [8], eBay [11], Groth [14], Library and Archives Canada [16], and Marlen [18], as well as in the *Scott* stamp catalogues [26] and *The Unitrade Specialized Catalogue of Canadian Stamps* [35]. Much of our biographical and geographical information is based on that in *Wikipedia* [36]. Research was supported, in part, by the Natural Sciences and Engineering Research Council of Canada.

## References

[1] Lars Døvling Andersen (2008). Chapter on "The history of Latin squares" intended for publication in *The History of Combinatorics* edited by Robin J. Wilson. Preprint: [Report] R-2007-32, Department of Mathematical Sciences, Aalborg University, Aalborg Øst, Denmark: online: 31 pp. 1, 2

[2] Rowan S. Baker. USA Stamps at The Covent Garden Stamp Shop, miniature sheets & special sheetlets: online open-access website in London, England. 8

[3] BiStamp.com. Stamp professionals specializing in postage stamps of Russia and other ex-USSR states and worldwide topicals: online open-access website at Kyiv, Ukraine. 8

[4] Ka Lok Chu, Simo Puntanen & George P. H. Styan (2009). Some comments on philatelic Latin squares from Pakistan. *Pakistan Journal of Statistics,* 25 (4), 427–471. 2

[5] Charles J. Colbourn & Jeffrey H. Dinitz, eds. (2007). *Handbook of Combinatorial Designs*, Second edition. Chapman & Hall/Taylor & Francis, Boca Raton. 1, 2

[6] [François] Cretté de Palluel (1788). Sur les avantages & l'économie que procurent les racines employées à l'engrais des moutons à l'étable. *Mémoires d'agriculture, d'économie rurale et domestique,* Trimestre d'été 1788 ("Lu le 31 juillet 1788"), pp. 17–23 (& 2 foldout-pages of tables facing p. 23). 2, 8

[7] [François] Cretté de Palluel (1790). On the advantage and economy of feeding sheep in the house with roots. *Annals of Agriculture and Other Useful Arts,* 14 (read July 31, 1788), 133–139 & foldout-pages facing p. 139. [English translation (with full data set) of [6]. For a "contemporary translation" into English (with full data set) see Street & Street [28, pp. 51–55].] 2

[8] delcampe.com. The international marketplace for [stamp] collectors: online open-access website in Portland, Oregon. 8

[9] J. Dénes & A. D. Keedwell (1974). *Latin Squares and their Applications*. Academic Press, New York & Akadémiai Kiadó, Budapest. 1

[10] J. Dénes & A. D. Keedwell, eds. (1991). *Latin Squares: New Developments in the Theory and Applications,* with specialist contributions by G. B. Belyavskaya, A. E. Brouwer, T. Evans, K. Heinrich, C. C. Lindner & D. A. Preece. North-Holland, Amsterdam. 9

[11] eBay.com. Open-access auction and shopping website: online for stamps. 8

[12] Leonhard Euler (1776). De quadratis magicis (in Latin). Talk delivered to the St. Petersburg Academy of Sciences on October 17, 1776, and first published in *Leonhardi Euleri Commentationes arithmeticae collectae, auspiciis Academiae imperialis scientiarum petropolitanae*, vol. 2 (1849), pp. 593–

602. [Eneström index no. E795. Reprinted in *Leonhardi Euleri Opera Omnia, Series I: Opera Mathematica,* vol. I.7, pp. 441–457. Online (with commentary and references) in *The Euler Archive* [13]. Translated from Latin into English as "On magic squares" by Jordan Bell, 4 December 2004.] 2, 9

[13] *The Euler Archive: The Works of Leonhard Euler Online.* Dartmouth College, Hanover, NH. [Includes [12].] 9

[14] Groth AG. WWF Conservation Stamp Collection: online open-access website in Unterägeri (canton Zug), Switzerland. 3, 8

[15] Charles F. Laywine & Gary L. Mullen (1998). *Discrete Mathematics using Latin Squares*. Wiley, New York. 2

[16] Library and Archives Canada (2008). Canadian Postal Archives Database: online open-access search by keyword or by year. 8

[17] Peter D. Loly & George P. H. Styan (2009). An introduction to philatelic Latin squares, with some comments on Sudoku. Report 2009-01 from the Department of Mathematics and Statistics, McGill University, Montréal, 59 pp., 27 September 2009. 8

[18] Marlen Stamp & Coins Ltd: online open-access website in Great Neck, New York. 8

[19] D. A. Preece (1990). R. A. Fisher and experimental design: a review. *Biometrics,* 46 (4), 925–935: online at JSTOR. 2

[20] D. A. Preece (1991). Latin squares as experimental designs. In Dénes & Keedwell [10, ch. 10, pp. 317–342]. 2

[21] Simo Puntanen & George P. H. Styan (2008). Stochastic stamps: a philatelic introduction to chance. *Chance*, 21 (3), 36–41: online at SpringerLink. 2

[22] Simo Puntanen & George P. H. Styan (2009). A philatelic introduction to statistical physics. *ESI-News*, 4 (1), 5–7: online open-access at The Erwin Schrödinger International Institute for Mathematical Physics (ESI), Vienna. 2

[23] Terry Ritter (1998–2003). Latin squares: a literature survey. Research comments from *Ciphers By Ritter:* online open-access, last updated 21 December 2003, 22 pp. 1

[24] H. S. Roberts, ed. (1999). *A History of Statistics in New Zealand,* edited and written by H. Stanley Roberts (b. 1920). New Zealand Statistical Association, Wellington. 8

[25] William L. Schaaf (1978). *Mathematics and Science: An Adventure in Postage Stamps.* National Council of Teachers of Mathematics, Reston, Virginia. 2

[26] *Scott Standard Postage Stamp Catalogue* (James E. Kloetzel, ed.), currently published annually by Scott Publishing, Sidney, Ohio. 8

[27] "Student" (1938). Comparison between balanced and random arrangements of field plots. *Biometrika,* 29 (3/4), 363–378: online at JSTOR. 8

[28] Anne Penfold Street & Deborah J. Street (1988). Latin squares and agriculture: the other bicentennial. *The Mathematical Scientist,* 13, 48–55. 2, 8

[29] George P. H. Styan (2007). A philatelic introduction to magic squares and Latin squares for Euler's 300th birthyear. In *Proceedings of the Canadian Society for History and Philosophy of Mathematics,* vol. 20, pp. 306–319. 1, 2

[30] George P. H. Styan (2009). An illustrated philatelic introduction to $4 \times 4$ Latin squares in Europe: 1283–1788 (with some comments about *le fauteuil Dagobert* and about philatelic Latin squares in Europe: 1984–2007). Talk presented at the 18th International Workshop on Matrices and Statistics: IWMS'09, Smolenice Castle, Smolenice, Slovakia, 26 June 2009, overheads pdf file, 28 pp. Talk based on Report 2009-02 from the Department of Mathematics and Statistics, McGill University, Montréal, 43 pp., 12 June 2009. 2

[31] George P. H. Styan (2009). Personal postage stamp collection. Verdun (QuŐbec), Canada, 24 November 2009.

[32] George P. H. Styan, Christian Boyer & Ka Lok Chu (2009). Some comments on Latin squares and on Graeco-Latin squares, illustrated with postage stamps and old playing cards. *Statistical Papers,* 50 (4), 917–941: online at SpringerLink. 1, 2

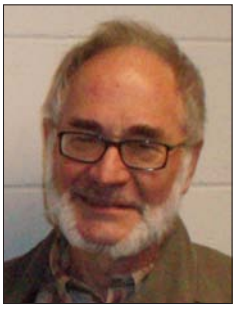
[33] Peter Ullrich (1999). An Eulerian square before Euler and an experimental design before R. A. Fisher: on the early history of Latin squares. *Chance,* 12 (1), 22–26. 2

[34] Peter Ullrich (2002). Officers, playing cards, and sheep: on the history of Eulerian squares and of the design of experiments. *Metrika,* 56 (3), 189–204: online at SpringerLink. 2

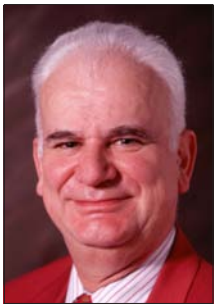
[35] *The Unitrade Specialized Catalogue of Canadian Stamps, 2009 Edition* (2008). Featuring the established Scott numbering system [and] including British Columbia and Vancouver Island, New Brunswick, Newfoundland, Nova Scotia and Prince Edward Island. The Unitrade Press, Toronto. 8

[36] *Wikipedia, The Free Encyclopedia.* Wikimedia Foundation: online open-access web archive. 2, 8

[37] F. Yates (1939). The comparative advantages of systematic and randomized arrangements in the design of agricultural and biological experiments. *Biometrika,* 30 (3/4), 440–466: online at JSTOR. 1, 8



**Peter D. Loly** is Senior Scholar and Professor, Department of Physics and Astronomy, The University of Manitoba, Winnipeg.



**George P. H. Styan** is Professor Emeritus, Department of Mathematics and Statistics, McGill University, Montréal. He was Editor of *Chance* magazine 1996–1998.